CSIR-JRF/NET (Dec. 2013)

MATHEMATICAL SCIENCE

Subject Code – 4 **Booklet Code – A**

2013 (I)

TEST BOOKLET

(22 Dec. 2013)

Time Allowed: Three Hours **Maximum Marks: 200**

INSTRUCTIONS

1. **You have opted for English as medium of Question Paper.** This Test Booklet contains one hundred and twenty (20 Part 'A' + 40 Part 'B' + 60 Part 'C') Multiple Choice Questions (MCQs). You are required to answer a maximum of 15, 25 and 20 questions from part 'A', 'B' and 'C' respectively. If more than required number of questions are answered, only first 15, 25 and 20 questions in Part 'A', 'B' and 'C' respectively, will be taken up for evaluation.
2. Answer sheet has been provided separately. Before you start filling up your particulars, please ensure that the booklet contains requisite number of pages and that these are not torn or mutilated. If it is so, you may request the Invigilator to change the booklet. Likewise, check the answer sheet also. Sheets for rough work have been appended to the test booklet.
3. Write your Roll No., Name, Your address and Serial Number and this Test Booklet on the Answer sheet in the space provided on the side 1 of Answer sheet. Also put your signatures in the space identified.
4. You must darken the appropriate circles related to Roll Number, Subject Code, Booklet Code and Centre Code on the OMR answer sheet. It is the sole responsibility of the candidate to meticulously follow the instructions given on the Answer Sheet, failing which, the computer shall not be able to decipher the correct details which may ultimately result in loss, including rejection of the OMR answer sheet.
5. Each question in Part 'A' carries 2 marks, Part 'B', 3 marks and Part 'C' 4.75 marks respectively. There will be negative marking @0.5 marks in Part 'A' and @0.75 in Part 'B' for each wrong answer and no negative marking for part 'C.
6. Below each question in Part 'A' and 'B', four alternatives or responses are given. Only one of these alternatives is the "correct" option to the question. You have to find, for each question, the correct or the best answer. In Part 'C' each question may have **'ONE'** or **'MORE'** correct options. Credit in a question shall be given only on identification of **'ALL'** the correct options in Part 'C'. No credit shall be allowed in a question if any incorrect option is marked as correct answer.
7. Candidates found copying or resorting to any unfair means are liable to be disqualified from this and future examinations.
8. Candidate should not write anything anywhere except on answer sheet or sheets for rough work.
9. After the test is over, you MUST hand over the answer sheet (OMR) to the invigilator.
10. Use of calculator is not permitted.

Roll No. …………………………..

Name ……………………………..

I have verified all the information

filled in by the candidate.

…………………………………………..

Signature of the Invigilator

CSIR-JRF/NET (Dec. 2013)

MATHEMATICAL SCIENCE

PART-A

(1.) A hemispherical bowl is being filled with water at constant volumetric rate. The level of water in the bowl increases

a.) in direct proportion to time,

b.) in inverse proportion to time,

c.) faster than direct proportion to time,

d.) slower than direct proportion to time.

(2.) Equal masses of two liquids of densities kg/m^3 and 4 kg/m^3 are missed thoroughly. The density of the mixture is

a.) 4.8 kg/m^3

b.) 5.0 kg/m^3

c.) 5.2 kg/m^3

d.) 5.4 kg/m^3

(3.) Two points A and B on the surface of the Earth have the following latitude and longitude co-ordinates.
A: 30° N, 45° E
B: 30° N, 135° W
If R is the radius of the Earth, the length of the shortest path from **A** to **B** is

a.) $\frac{\sqrt{3}}{2}\pi R$

b.) $\frac{\pi R}{3}$

c.) $\frac{\pi R}{6}$

d.) $\frac{2\pi R}{3}$

(4.) Amoebae are known to double in 3 min. Two identical vessels A & B, respectively contain one and two amoebae to start with. The vessel B gets filled in 3 hours. When will A get filled?

a.) 3 hours

b.) 2 hours 57 min

c.) 3 hours 3 min

d.) 6 hours

(5.) Students of a school are divided into 4 groups. What is the probability that three friends get into the same group?

a.) $\frac{3}{4}$

b.) $\frac{1}{64}$

c.) $\frac{1}{16}$

d.) $\frac{1}{3}$

(6.) A fruit vendor buys 120 Shimla apples at 4 for Rs. 100, and 120 Golden apples at 6 for Rs. 100. She decides to mix them and sell at 10 for Rs. 200, She will make

a.) no profit, no loss

b.) a loss of 4%

c.) a gain of 4%

d.) a loss of 10%

(7.) $4^0 + 4^2 + 4^{-2} + 4^{1/2} + 4^{-1/2} =$

a.) 4^0

b.) $4^{2\frac{1}{2}} + 4^{-2\frac{1}{2}}$

c.) $19\frac{9}{16}$

d.) $22\frac{9}{16}$

(8.) In an enclosure there were both crows and cows. If there were 30 heads and 100 legs, what fraction of them are crows?

a.) 1/3

b.) 1/4

c.) 1/10

d.) 3/10

(9.) A cart wheel rolls along a straight line. If the distance covered is equal to the diameter of the wheel, what is the angle through which the wheel has turned?

a.) 90°

b.) between 90° and 120°

c.) between 120° and 150°

d.) between 150° and 180°

(10.) In a class of 10 students, 3 failed in **13**. History, 6 failed in Geography and 2 failed in both. How many passed in both the subject?

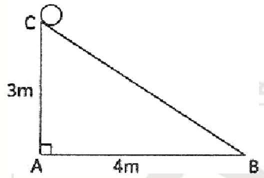
a.) 1

b.) 2

c.) 3

d.) 0

(11.)



As shown in the diagram above, a sphere is placed on the top of an incline. It rolls down the incline without slipping in exactly 50. The radius of the sphere is

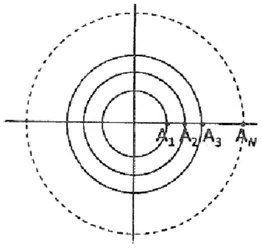
a.) $\left(\frac{5}{\pi}\right)$cm

b.) $\left(\frac{5}{\pi}\right)$m

c.) $\left(\frac{10}{\pi}\right)$cm

d.) 10 cm

(12.)



A set of concentric circles of integer radii 1,2, $\cdots N$ is shown in the figure above. An ant starts at point A_1, goes round the first circle, returns to A_1, moves to A_2, goes round the second circle, returns to A_2, moves to A_3 and repeats this until it reaches A_N. The distance covered by the ant is

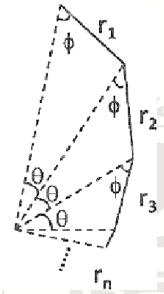
a.) $N(N+1)\pi$

b.) $2N\pi+N$

c.) $\pi(N+1)N+N-1$

d.) $\pi(N-1)N+N-1$

(13.)



The figure above shows an infinite series of triangles, in which $r_1 > r_2 > r_3 \ldots$ What is the total length of the solid line segments in the figure?

a.) $\frac{r_1}{r_2}+\frac{r_2}{r_3}+\cdots$

b.) $\frac{r_1}{r_1-r_2}$

c.) $\frac{r_1^2}{r_1+r_2}$

d.) $\frac{r_1-r_2}{r_1^2}$

(14.) If a_i, b_i and c_i are distinct, how many terms will the expansion of the product $(a_1+a_2+a_3)(b_1+b_2+b_3+b_4)(c_1+c_2+c_3+c_4+c_5)$ contain?

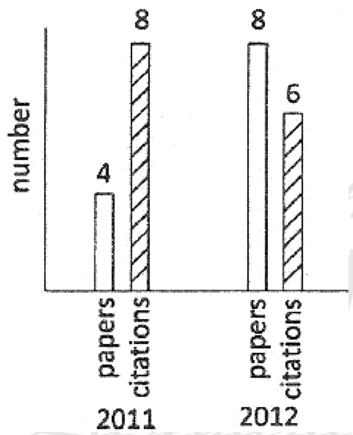
a.) 12

b.) 30

c.) 23

d.) 60

(15.)



The above plot depicts the number of research publications of a scientist along with the number of citations. Which of the following statements is **not** correct?

a.) In the year 2012, 50% more papers were published but citations decreased by 25%.

b.) In the year 2012, 100% more papers were published but citations were 75% of the number of papers in that year.

c.) The papers published in year 2011 is only 33.33% of the total number of papers in both years.

d.) The total number of citations for both years is 16.66% more than the total number of papers.

5.

(16.) The next number of the sequence 1, 5, 14, 30, 55, . . . is

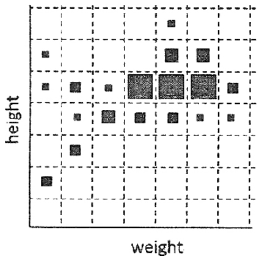
a.) 85

b.) 90

c.) 91

d.) 95

(17.)



The distribution of heights and weights in a population is shown above in a 2- parameter scatter plot. The size of the square is proportional to the number of persons having a particular combination of weight and height. Which statement best describes the trend in the population?

a.) Height and weight are strongly correlated.

b.) Height and weight are anticorrelated.

c.) Large heights do not imply proportionately large weights.

d.) Height and weight are independent characteristics.

(18.) What is the maximum sum of the numbers of Saturdays and Sundays in a leap year?

a.) 104

b.) 105

c.) 106

d.) 107

(19.) Two trains of lengths 150 m and 250 m pass each other with constant speeds on parallel tracks in opposite directions. The drivers and guards are at the extremities of the trains. The time gap between the drivers passing each other and first driver-guard pair passing each other is 30 s. How much later will the other driver-guard pair pass by?

a.) 10 s

b.) 20 s

c.) 30 s

d.) 50 s

(20.) In a room, we have one grandfather, two fathers, two sons, and grandson. The age of one father is seven times the age of this son. The age of the other father is twice his son’s age. Assuming that there are only 3 people in the room and the grandfather is 70 years old, how old is the grandson?

a.) 1

b.) 2

c.) 5

d.) Cannot be determined

PART-B

(21.) Let f be a non constant entire function. Which of the following properties is possible for f fir each $z \in \mathbb{C}$?

a.) $\operatorname{Re} f(z) = \operatorname{Im} f(z)$.

b.) $|f(z)| < 1$.

c.) $\operatorname{Im} f(z) < 0$.

d.) $f(z) \neq 0$.

(22.) If $f : [0, 1] \to (0, 1)$ is a continuous mapping then which of the following is NOT ture?

a.) $F \subseteq [0, 1]$ is a closed set implies $f(F)$ is closed in $\mathbb{R}$.

b.) If $f(0) < f(1)$ then $f([0, 1])$ must be equal to $[f(0), f(1)]$.

c.) There must exist $x \in (0, 1)$ such that $f(x) = x$.

d.) $f : ([0, 1]) \neq (0, 1)$.

(23.) Let a, b, c be non-collinear points in the complex plane and let Δ denote the closed triangular region of the plane with vertices a, b, c. For $z \in \Delta$, let $\mathrm{h}(z) = |z-a| \cdot |z-b| \cdot |z-c|$. The maximum value of the function h:

a.) is not attained at any point of Δ.

b.) is attained at an interior point of Δ.

c.) is attained at the centre of gravity of Δ.

d.) is attained at a boundary point of Δ.

(24.) Let f be a nonconstant holomorphic function in the unit disc $\{|z| < 1\}$ such that $f(0) = 1$. Then it is necessary that

a.) there are infinitely many points z in the unit disc such that $|f(z)| = 1$.

b.) f is bounded.

c.) there are at most finitely many points z in the unit disc such that $|f(z)| = 1$.

d.) f is a rational function.

(25.) How many normal subgroups does a non-abelian group G of order 21 have other that the identity subgroup $\{e\}$ and G?

a.) 0.

b.) 1.

c.) 3.

d.) 7.

(26.) For any integers a, b let $N_{a,b}$ denote the number of positive integers $x < 1000$ satisfying $x \equiv a \ (\bmod 27)$ and $x \equiv b (\bmod 37)$. Then,

a.) there exist a, b such that $N_{a,b} = 0$.

b.) for all $a, b, N_{a,b} = 1$.

c.) $a, b, N_{a,b} > 1$

d.) there exist a, b such that $N_{a,b}a = 1,$ and there exist a, b such that $N_{a,b} = 2$.

(27.) Let τ_1 be the product (standard) topology on $\mathbb{R}^2$ generated by the base
$B_1 = \{(s, t) \times (u, v) : s < t,$
$u < v$ where $s, r, u, v \in R\}$
(B_1 is the collection of product of open intervals.) Given $r, R \in R$ with $0 < r < R$ and $a = (a_1, a_2) \in R^2$, let
$c(a,r,R) = \{(x_1, x_2) \in R^2 :$
$r^2 < (x_1 - a_1)^2 + (x_2 - a_2)^2 < R^2\}$.
Let
$B_2 = \{C(a,r,R) : a \in \mathbb{R}^2,\ r, R \in \mathbb{R}, 0 < r < R\}$.
Let τ_2 be the topology generated by the base B_2.
Then

a.) $\tau_1 \subseteq \tau_2, \tau_1 \neq \tau_2$.

b.) $\tau_2 \subseteq \tau_1, \tau_1 \neq \tau_2$

c.) $\tau_1 \subseteq \tau_2$

d.) $\tau_1 \not\subseteq \tau_2, \tau_2 \not\subseteq \tau_1$.

(28.) The number of group homomorphisms from the symmetric group S_3 to the additive group $\mathbb{Z}/6\mathbb{Z}$ is a.) 1.

b.) 2.

c.) 3.

d.) 0.

(29.) In a 2×2 contingency table if we multiply a particular column by an integer $k(>1)$, then the odds ratio

a.) will increase.

b.) will decrease.

c.) remains same.

d.) will increase if $k > 2$ and will decrease if $k = 2$.

(30.) A popular car comes in both a petrol and diesel version. Each of these is further available in 3 models, L, V and Z. Among all owners of the petrol version of this car 50% have model V and 20% have model Z. Among diesel car customers, 50% have model L and 20% model V. 60% of all customers have bought diesel cars. If a randomly chosen customer has model V, what is the probability that the car is diesel car?

a.) 3/8.

b.) 3/5.

c.) 1/5.

d.) 2/3.

(31.) Let $X_1, X_2, \ldots$ be a Markov chain with state space $\{1,2,3,4\}$. Let the transition probability matrix p be given by

$$p = \begin{pmatrix} 1/3 & 0 & 0 & 2/3 \\ 1/4 & 1/4 & 1/4 & 1/4 \\ 0 & 0 & 1 & 0 \\ 2/3 & 0 & 0 & 1/3 \end{pmatrix}$$

Which of the following is a stationary distribution for the Markov chain?

a.) $(1/4 \quad 1/4 \quad 1/47 \quad 1/4)$

b.) $(1/3 \quad 0 \quad 0 \quad 2/3)$.

c.) $(0 \quad 1/4 \quad 1/2 \quad 1/4)$.

d.) $(1/3 \quad 0 \quad 1/3 \quad 1/3)$

(32.) Let $X_1, X_2,..., X_n$ be a random sample from uniform $\left(\theta-\frac{1}{2}, \theta+\frac{1}{2}\right)$. Consider the problem of testing $H_0 : \theta = -\frac{1}{2}$ against $H_1 : \theta = -\frac{1}{2}$. Define $X_{(1)} = \min\{X_1, X_1,..., X_n\}$. Consider the following test:

Reject H_0 if $X_{(1)} > 0$, accept otherwise. Which of the following is true?

a.) power of the test $=0$, size of the test $=0$.

b.) power of the test $=0$, size of the test $=1$.

c.) power of the test $=1$, size of the test $=0$.

d.) power of the test $=1$, size of the test $=1$.

(33.) Suppose the cumulative distribution function of failure time T of a component is

$$1-\exp\left(-ct^{\alpha}\right), \qquad t>0,\ \alpha>1,\ c>0$$

Then the hazard rate of $\lambda(t)$ is

a.) constant.

b.) non-constant monotonically increasing in t.

c.) non-constant monotonically decreasing t.

d.) not a monotone function in t.

(34.) A factorial experiment involving 4 factors F_1, F_2, F_3 and F_4 each at 2 levels, 0 and 1, is planned in 4 blocks each of size 4. One of the these blocks has the following contents:

F_1	F_2	F_3	F_4
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

The confounded factorial effect are

a.) $F_1F_2,\ F_1F_3,\ F_2F_3$

b.) $F_1F_3,\ F_1F_2F_4,\ F_2F_3\,F_4$

c.) $F_3F_4,\ F_1F_2F_3,\ F_1F_2\,F_4$

d.) $F_1F_4,\ F_2F_3,\ F_1F_2\,F_3\,F_4$

(35.) Let $X_0, \in_1, \in_2, \in_3$ and $\in_4$ be independent and identically distributed normal random variables with mean 0 and variance $\sigma^2 > 0$. Define $X_{i+1} = \rho X_i + \left(\sqrt{1-\rho^2}\right) \in_{i+1}$,
where $0 < \rho < 1$, for $i = 0,\ 1,\ 2,\ 3.$ Let $\rho_{ij\cdot k}$ denote the partial correlation between X_i and X_j given X_k. Then $\rho_{14.2} =$

a.) ρ^3.

b.) ρ^2.

c.) ρ.

d.) 0.

(36.) Consider the following probability mass function $P_{\theta_1,\theta_2}(x)$ where the parameters (θ_1, θ_2) take values in the parameter space
$\left\{\left(\frac{1}{3},3\right),\left(\frac{1}{2},2\right),\left(2,\frac{1}{2}\right),\left(3,\frac{1}{3}\right)\right\}$:

(θ_1, θ_2) x	$\left(\frac{1}{3}, 3\right)$	$\left(\frac{1}{2}, 2\right)$	$\left(2, \frac{1}{2}\right)$	$\left(3, \frac{1}{3}\right)$
1	$1/11$	$1/7$	$1/8$	$1/9$
2	$1/11$	$1/14$	$1/16$	$1/9$
3	$8/11$	$5/7$	$3/4$	$2/3$
4	$1/11$	$1/14$	$1/16$	$1/9$

Let X be a random observation from this distribution. If the observed value of X is 3, then

a.) MLE of $\theta_1 = 1/3$, MLE of $\theta_2 = 3$

b.) MLE of $\theta_1 = 1/2$, MLE of $\theta_2 = 2$.

c.) MLE of $\theta_1 = 2$, MLE of $\theta_2 = 1/2$.

d.) MLE of $\theta_1 = 3$, MLE of $\theta_2 = 1/3$.

(37.) Suppose $D \sim N(0, 1)$ and

$$U = \begin{cases} 1 & if\ D \geq 0 \\ 0 & if\ D < 0. \end{cases}$$

Then the correlation coefficient between $|D|$ and U is equal to

a.) 0.5.

b.) 0.25.

c.) 1.

d.) 0.

(38.) Suppose $X_{1,}X_2,...,X_n$ are independent and identically distributed random variables each giving an exponential distribution with parameter $\lambda > 0$. Let $X_{(1)} \le ... \le X_{(n)}$ be the correstponding order statistics. Then the probability distribution of $(X_{(n)} - X_{(n-1)})/nX_{(1)}$ is

a.) Chi-square with 1 degree of freedom.

b.) Beta with parameters 2 and 1.

c.) F with parameters 2 and 2.

d.) F with parameters 2 and 1.

(39.) A population contains three units u_1, u_2 and u_3. For $i = 1, 2, 3,$ let Y_i be the value of a study variable for u_i. A simple random sample of size two is drawn from the population without replacement. Let T_1 denote the usual sample mean and let T_2 and T_3 be two other estimators, defined as follows:

$$T_2 = \begin{cases} \frac{1}{2}(Y_1 + Y_2) & \text{if } u_1, u_2 \text{ ara in the sample} \\ \frac{1}{2}\left(Y_1 + \frac{2}{3}Y_3\right) & \text{if } u_1, u_3 \text{ ara in the sample} \\ \frac{1}{2}Y_2 + \frac{1}{3}Y_3 & \text{if } u_2, u_3 \text{ ara in the sample} \end{cases}$$

$$T_3 = \begin{cases} \frac{1}{2}(Y_1 + Y_2) & \text{if } u_1, u_2 \text{ ara in the sample} \\ Y_1 + \frac{1}{2}Y_3 & \text{if } u_1, u_3 \text{ ara in the sample} \\ \frac{1}{2}Y_2 + \frac{1}{3}Y_3 & \text{if } u_2, u_3 \text{ ara in the sample} \end{cases}$$

If $\bar{Y}$ is the population mean, then which of the following statements is true?

a.) All the three estimators $T_1,\ T_2,\ T_3$ are unbiased for $\bar{Y}$.

b.) T_2 and T_3 are biased estimator of $\bar{Y}$ but T_1 is not.

c.) T_1 and T_2 are unbiased for $\bar{Y}$ but T_3 is not.

d.) T_1 and T_3 are unbiased for $\bar{Y}$ but T_2 is not.

(40.) Let $X_1, X_2,...,X_n$ be a random sample form $N\left(\theta, \sigma^2\right)$ where $\sigma^2 > 0$ is known. Suppose θ has the Cauchy prior with density

$$\frac{1}{n}\left(1 + \left(\frac{\theta - \mu}{\tau}\right)^2\right)^{-1}, -\infty < \theta < \infty,$$

with μ and τ known. Then with reference to the posterior distribution of θ

a.) the posterior mean does not exist and the posterior variance is ∞.

b.) the posterior mean exists but the posterior variance is ∞.

c.) the posterior mean exists and the posterior variance is finite.

d.) the posterior variance is finite but the posterior mean does not exist.

(41.) Let W be the Wronskian of two linearly independent solutions of ODE

$2y''+y'+t^2y=0; t\in\mathbb{R}$

Then, for all t, there exists a constant $C\in\mathbb{R}$ such that $W(t)$ is

a.) Ce^{-1}.

b.) $Ce^{-t/2}$.

c.) Ce^{2t}.

d.) Ce^{-2t}.

(42.) Let a, b, c be continuous functions defined on $\mathbb{R}^2$. Let V_1, V_2, V_3 be nonempty subset of $\mathbb{R}^2$ such that $V_1 \cup V_2 \cup V_3 = \mathbb{R}^2$ and the PDE $a(x, y)u_{xx} + b(x, y)u_{xy} + c(x, y)u_{yy} = 0$ is elliptic in V_1, parabolic in V_2 and hyperbolic in V_3, then

a.) V_1, V_2 and V_3 are open sets in $\mathbb{R}^2$

b.) V_1, and V_3 are open sets in $\mathbb{R}^2$

c.) V_1, and V_2 are open sets in $\mathbb{R}^2$

d.) V_2, and V_3 are open sets in $\mathbb{R}^2$

(43.) The partial differential equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + u$$

can be transformed to

$$\frac{\partial v}{\partial t} = \frac{\partial^2 v}{\partial x^2}$$

For

a.) $v = e^{-t}u.$

b.) $v = e^{t}u.$

c.) $v = tu$.

d.) $v = -tu$

(44.) Let $G(x,\ y)$ be the Green's function of the boundary value problem $[(1+x)u']' + (\sin x)u = 0,\ x \in [0,1]\, u(0) = u(1) = 0$.

Then the function g defined by

$$g(x) = G\left(x, \frac{1}{2}\right),\ x \in [0,1]$$

a.) is continuous.

b.) is discontinuous at $x = \frac{1}{2}$.

c.) is differentiable.

d.) does not have the left derivative at $x = \frac{1}{2}$.

(45.) The integral equation

$$\varphi(x) = f(x) + \int_0^1 K(x,\ y)\varphi(y)\,dy$$

For $K(x,\ y) = xy^2$ has a solution

a.) $\varphi(x) = f(x)$.

b.) $\varphi(x) = f(x,\ x)$.

c.) $\varphi(x) = x^3$

d.) $\varphi(x) = f(x) + \frac{4}{3}x\int_0^1 x^2 f(x)\,dx$.

(46.) Consider the functional

$$J(y) = \int_a^b F(x, y, y')\,dx$$

where

$$F(x, y, y') = y' + y$$

For admissible functions y. Then J has

a.) no extremals.

b.) several extremals.

c.) $y(x) = e^{-x}$ as an extremal.

d.) $y(x) =$ constant as an extremal.

(47.) Let S be a mechanical system with Lagrangian $L(\vec{q}, \dot{\vec{q}}, t)$ and generalized coordinates $\vec{q} = (q_1, q_2, \ldots, q_n)$. Then the Lagrange equations of motion for S

a.) constitute a set of n first order ODEs.

b.) can be transformed to the Hamilton form using Legendre transform.

c.) are equivalent to se of n first order ODEs when expressed in terms of Hamiltonian functions.

d.) is a set of $2n$ second order ODEs.

(48.) Consider the initial value problem

$$\frac{dy}{dx} = x + y,\ y(0) = 1.$$

Then the approximate value of the solution $y(x)$ at $x = 0.2,$ using improved Euler method, with $h = 0.2$ is

a.) 1.11.

b.) 1.20.

c.) 1.24.

d.) 1.48.

(49.) Let $f_n : [1, 2] \to [0, 1]$ be given by $f_n(x) = (2 - x)^n$ for all non-negative integers n.

Let $f(x) = \lim_{n \to \infty} f_n(x)$ for $1 \le x \le 2$. Then which of the following is true?

a.) f is a continuous function on $[1, 2]$.

b.) f_n converges uniformly to f on $[1, 2]$ as $n \to \infty$.

c.) $\lim_{n \to \infty} \int_1^2 f_n(x)\,dx = \int_1^2 f(x)\,dx$.

d.) for any $a \in (1, 2)$ we have $\lim_{n \to \infty} f'_n(a) \ne f'(a)$.

(50.) For a fixed positive integer $n \ge 3,$ let A be the $n \times n$ matrix defined as $A = I - \frac{1}{n}J$, where I is the identity matrix and J is the $n \times n$ matrix with all entries equal to 1. Which of the following statements is NOT true?

a.) $A^k = A$ for every positive integer k.

b.) Trace $(A) = n-1$

c.) Rank $(A)+$Rank $(I-A)=n$.

d.) A is invertible.

(51.) Let A be a 5×4 matrix with real entries such that $A\underline{x}=\underline{0}$ if and only if $\underline{x}=\underline{0}$ where $\underline{x}$ is a 4×1 vector and $\underline{0}$ is a null vector. Then, the rank of A is

a.) 4.

b.) 5.

c.) 2.

d.) 1.

(52.) Consider the following row vectors :

$$a_1 = (1,\ 1,\ 0,\ 1,\ 0,\ 0),\quad a_2 = (1,\ 1,\ 0,\ 0,\ 1,\ 0)\quad a_2 = (1,\ 1,\ 0,\ 0,\ 0,\ 1),$$
$$a_4 = (1,\ 0,\ 1,\ 1,\ 0,\ 0),\quad a_5 = (1,\ 0,\ 1,\ 0,\ 1,\ 0)\quad a_6 = (1,\ 0,\ 1,\ 0,\ 0,\ 1),$$

The dimension of the vector space spanned by these row vector is

a.) 6.

b.) 5.

c.) 4.

d.) 3.

(53.) Let $A_n \times n = \left(\left(a_{ij}\right)\right),\ n \ge 3,$ where $a_{ij} = \left(b_i^2 - b_j^2\right), i, j = 1, 2, \cdots, n$ for some distinct real numbers $b_1, b_2, \cdots, b_n$. Then $\det(A)$ is

a.) $\prod_{i<j}\left(b_i - b_j\right)$.

b.) $\prod_{i<j}\left(b_i + b_j\right)$

c.) 0.

d.) 1.

(54.) Let $\{a_n\},\ \{b_n\}$ be sequences of real numbers satisfying $|a_n| \le |b_n|$ for all $n \ge 1$. Then

a.) $\sum_n a_n$ converges whenever $\sum_n b_n$ converges.

b.) $\sum_n a_n$ converges absolutely whenever $\sum_n b_n$ converges absolutely.

c.) $\sum_n b_n$ converges whenever $\sum_n a_n$ converges.

d.) $\sum_n b_n$ converges absolutely whenever $\sum_n a_n$ converges absolutely.

(55.) If $\sum_{n=1}^{\infty} a_n$ is absolutely convergent, then which of the following is NOT ture?

a.) $\sum_{m=n}^{\infty} a_m \to 0$ as $n \to \infty$.

b.) $\sum_{n=1}^{\infty} a_n \sin n$ is convergent.

c.) $\sum_{n=1}^{\infty} e^{a_n}$ is divergent.

d.) $\sum_{n=1}^{\infty} a_n^2$ is divergent.

(56.) Let $\lambda > 0$ and $F(x) = 1 - e^{-\lambda x}$ for $x > 0$. Then for $t > 0$, $\int_0^\infty e^{-tx} dF(x)$ equals

a.) $\frac{\lambda}{\lambda + t}$

b.) $\frac{\lambda}{\lambda - t}$

c.) 0.

d.) ∞.

(57.) Let

$$f(x) = \begin{cases} \frac{\sin}{x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0. \end{cases}$$

Then f is

a.) discontinuous.

b.) continuous but not differentiable.

c.) differentiable only once.

d.) differentiable more than once.

(58.) Let $f:[0,1]\to[0,1]$ be any twice differentiable function satisfying $f\left(ax+(1-a)y\right)\le af(x)+(1-a)f(y)$ for all $x,y\in[0,1]$ and any $a\in[0,1]$. Then for all $x\in(0,1)$

a.) $f'(x)\ge 0$.

b.) $f''(x)\ge 0$.

c.) $f'(x)\le 0$.

d.) $f''(x)\le 0$.

(59.) Let A be an $n\times n$ matrix with real entries. which of the following is correct?

a.) If $A^2=0$, then A is diagonalizable over complex numbers.

b.) If $A^2=I$, then A is diagonalizable over real numbers.

c.) If $A^2=A$, then A is diagonalizable only over complex numbers.

d.) The only matrix of size n satisfying the characteristic polynomial of A is A.

(60.) Let A be a 4×4 invertible real matrix. Which of the following is NOT necessarily true?

a.) The rows of A form a basis of $\mathbb{R}^4$.

b.) Null space of A contains only the 0 vector.

c.) A has 4 distinct eigenvalues.

d.) Image of the linear transformation $x\mapsto Ax$ on $\mathbb{R}^4$ is $\mathbb{R}^4$.

(61.) Let $L=\int_0^1\frac{dx}{1+x^8}$. Then

a.) $L<1$

b.) $L>1$

c.) $L<\frac{\pi}{4}$

d.) $L>\frac{\pi}{4}$

(62.) Let f,g be measurable real-valued functions on $\mathbb{R}$, such that

$$\int_{-\infty}^{\infty}\left(f(x)^2+g(x)^2\right)dx=2\int_{-\infty}^{\infty}f(x)g(x)\,dx$$

Let $E=\{x\in\mathbb{R} \mid f(x)\neq g(x)\}$. Which of the following statements are necessarily true?

a.) E is the empty set.

b.) E is measurable.

c.) E has Lebesgue mesure zero.

d.) For almost all $x\in\mathbb{R}$, we have $f(x)=0$ and $g(x)=0$.

(63.) For a continuous function $f:\mathbb{R}\to\mathbb{R}$ satisfying $\int_{\mathbb{R}}|f(x)|dx<\infty$ and for some $\alpha>0$, let $d_f(\alpha)$ be the Lebesgue measure of the set $\{x\in\mathbb{R} \mid |f(x)|>\alpha\}$.

Then, for all $\alpha>0$, we have

a.) $\alpha d_f(\alpha)\le\int_{\mathbb{R}}|f(x)|dx$.

b.) $\alpha^2 d_f(\alpha)\le\int_{\mathbb{R}}|f(x)|dx$.

c.) $d_f(a)\le\alpha\int_{\mathbb{R}}|f(x)|dx$.

d.) $d_f(a)\le\alpha^2\int_{\mathbb{R}}|f(x)|dx$.

(64.) Let $C(a,r)$ be the subset of $\mathbb{R}^2$ given by $C(a,r)=\{(x,y)\in\mathbb{R}^2 \mid (x-a)^2+y^2=r^2\}$.

Which of the following subsets of $\mathbb{R}^2$ are connected?

a.) $C(0,1)\cup C(0,2)$.

b.) $C(0,1)\cup C(1,3)$.

c.) $C(0,1)\cup C(1,1)$

d.) $C(0,1)\cup C(2,1)$

(65.) Let $(X,\|\cdot\|)$ be the normed linear space consisting of sequences $\alpha=\{\alpha(n)\}_{n=1}^{\infty}$ such that the series $\sum_{n=1}^{\infty}a(n)$ is absolutely convergent, with $\|a\|=\sum_{n=1}^{\infty}|a(n)|$. Let e_k denote the sequence in X whose $k-th$ term is 1 and other terms are 0's and let

$$E=\{e_k \mid k\in\mathbb{N}\}.$$

Then

a.) X is complete in the norm $\|\cdot\|$.

b.) E is bounded subset of X.

c.) E is a closed subset of X.

d.) E is a compact subset of X.

(66.) Let $f:\mathbb{R}^2 \to \mathbb{R}^2$ be given by

$$f(x, y) = (x+y, xy).$$

Then

a.) f is not differentiable at the point $(0, 0)$.

b.) The derivative of f is invertible except on the set $\{(x,y)\in\mathbb{R}^2 \mid x=y\}$.

c.) The inverse image of each point in $\mathbb{R}^2$ under f has at most two elements.

d.) f is surjective.

(67.) Let T_1, T_2 be two linear transformations form $\mathbb{R}^n$ to $\mathbb{R}^n$. Let $\{x_{1,}\ x_2, \cdots, x_n\}$ be a basis of $\mathbb{R}^n$. Suppose that $T_1 x_i \neq 0$ for every $i = 1, 2, \cdots, n$ and that $x_i \perp$ Ker T_2 for every $i = 1, 2, \cdots, n$. Which of the following is/are necessarily true?

a.) T_1 is invertible.

b.) T_2 is invertible.

c.) Both T_1, T_2 are invertible.

d.) Neither T_1 nor T_2 is invertible.

(68.) Let $S:\mathbb{R}^n \to \mathbb{R}^n$ be given by $v \mapsto \alpha v$ for a fixed $\alpha \in \mathbb{R}, \alpha \neq 0$. Let $T:\mathbb{R}^n \to \mathbb{R}^n$ be a linear transformation such that BB $= \{v_1, \cdots, v_n\}$ is a set of linearly independent eigenvectors of T .Then

a.) The matrix of T with respect to BB is diagonal.

b.) The matrix of $T - S$ with respect to BB is diagonal.

c.) The matrix of T with respect to BB is not necessarily diagonal, but upper triangular.

d.) The matrix of T with respect to BB is diagonal but the matrix of $(T - S)$ with respect to BB in not diagonal.

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(69.) For an $n\times n$ real matrix $A,\ \lambda\in\mathbb{R}$ and a nonzero vector $v\in\mathbb{R}^n$ suppose that $(A-\lambda I)^k v=0$ for some positive integer k. Let I be the $n\times n$ identity matrix. The n which of the following is/are always true?

a.) $(A-\lambda I)^{k+r} v=0$ for all positive integers r.

b.) $(A-\lambda I)^{k-1} v=0$.

c.) $(A-\lambda I)$ is not injective.

d.) λ is an eigenvalue of A.

(70.) Let y be a nonzero vector in an inner product space V. Then which of the following are subspaces of V?

a.) $\{x\in V\,|\,\langle x,\,y\rangle=0\}$.

b.) $\{x\in V\,|\,\langle x,\,y\rangle=1\}$.

c.) $\{x\in V\,|\,\langle x,\,y\rangle=0$ for all z such that $\langle z,\,y\rangle=0\}$.

d.) $\{x\in V\,|\,\langle x,\,z\rangle=1$ for all z such that $\langle z,\,y\rangle=1\}$.

(71.) The function $f:\mathbb{R}\to\mathbb{R}$ is given by $f(x)=e^{|x|+x^2}+|x^2-1|$

Which of the following is true about the function f?

a.) It is not differentiable exactly at three points of $\mathbb{R}$.

b.) It is not differentiable at $x=0$.

c.) It is differentiable at $x=2$.

d.) It is not differentiable at $x=1$ and $x=-1$.

(72.) Consider the sequence of rational number $\{q_k\}_{k\ge1}$ where

$$q_k=\sum_{n=1}^{k}\frac{1}{10^{n^2}}$$

i.e., the sequence is

$q_1=.1,\ q_2=.1001,\ q_3=.100100001$ etc.

Which of the following is true?

a.) This sequence is bounded and convergent in $\mathbb{Q}$.

b.) This sequence is not bounded.

c.) This sequence is bounded, but not a Cauchy sequence.

d.) This sequence is bounded and Cauchy but not convergent in $\mathbb{Q}$.

(73.) Which of the following subset of $\mathbb{R}^2$ are uncountable?

a.) $\{(a, b) \in \mathbb{R}^2 \mid a \le b\}$.

b.) $\{(a, b) \in \mathbb{R}^2 \mid a + b \in \mathbb{Q}\}$.

c.) $\{(a, b) \in \mathbb{R}^2 \mid ab \in \mathbb{Z}\}$.

d.) $\{(a, b) \in \mathbb{R}^2 \mid a, b \in \mathbb{Q}\}$

(74.) Let $\{a_n\}_{n \ge 1}$ be a sequence of positive numbers such that

$$a_1 > a_2 > a_3 > \cdots$$

Then which of the following is/are always true?

a.) $\lim_{n\to\infty} a_n = 0$.

b.) $\lim_{n\to\infty} \frac{a_n}{n} = 0$.

c.) $\sum_{n=1}^{\infty} \frac{a_n}{n}$ converges.

d.) $\sum_{n=1}^{\infty} \frac{a_n}{n^2}$ converges.

(75.) Let $\{v_1, \cdots, v_n\}$ be a linearly independent subset of a vector space V where $n \ge 4$. Set $w_{ij} = v_i - v_j$. Let W be the span of $\{w_{ij} \mid 1 \le i, j \le n\}$. Then

a.) $\{w_{ij} \mid 1 \le i < j \le n\}$ spans W.

b.) $\{w_{ij} \mid 1 \le i < j \le n\}$ is a linearly independent subset of W.

c.) $\{w_{ij} \mid 1 \le i \le n-1, j = i+1\}$ spans W.

d.) $\dim W = n$.

(76.) For any real square matrix M let $\lambda^{+}(M)$ be the number of positive eigenvalues of M counting multiplicities. Let A be an $n \times n$ real symmetric matrix and Q be an $n \times n$ real invertible matrix. Then

a.) Rank $A =$ Rank $Q^{T}AQ$.

b.) Rank $A =$ Rank $Q^{-1}AQ$.

c.) $\lambda^{+}(A) = \lambda^{+}\left(Q^{T}AQ\right)$.

d.) $\lambda^{+}(A) = \lambda^{+}\left(Q^{-1}AQ\right)$.

(77.) Let $f : \mathbb{R}^{m} \to \mathbb{R}^{m}$ be a differential function. Let $Df(x)$ be the derivative of f at $x \in \mathbb{R}^{m}$. Which of the following is/are correct?

a.) $Df(0)(u) = 0$ for all u in $\mathbb{R}^{m}$.

b.) $Df(0)(u) = 0$ for all u in $\mathbb{R}^{m}$ and some $x \in \mathbb{R}^{m}$ only if f is a constant.

c.) $Df(0)(u) = 0$ for all $u \in \mathbb{R}^{m}$ and all $x \in \mathbb{R}^{m}$ only if f is a constant.

d.) If f is not a constant function, then $Df(x)$ is a one to one function for some $x \in \mathbb{R}^{m}$.

(78.) If $f : S \to S$ is a function, then we denote by f^{k}, the function $f \circ f \circ \cdots \circ f$ $(k$ times$)$. Let f_1 and f_2 be two functions defined on $\mathbb{R}^{2}$ as follows

$f_1(x, y) = (x+1, y+3)$,

$f_2(x, y) = (x-3, y-2)$.

Then

a.) For any positive integer k, there exists a unique $(a, b) \in \mathbb{R}^{2}$, such that $f_1^{k}(0, 0) = f_2^{k}(a, b)$.

b.) For any real number a and any positive integer, there is at most one solution y for $f_1^{k}(0, 0) = f_2^{k}(a, y)$

c.) There exists $(a, b) \in \mathbb{R}^{2}$ such that $f_1^{k}(a, b) \neq f_2^{k}(x, y)$ for any $(x, y) \in \mathbb{R}^{2}$ and any positive integer k.

d.) f_1 is linear transformation.

(79.) Let f be a holomorphic function on the unit disc $\{|z| < 1\}$ in the complex plane. Which of the following is/are necessarily true?

a.) If for each positive integer n we have $f\left(\frac{1}{n}\right) = \frac{1}{n^{2}}$ then $f(z) = z^{2}$ on the unit disc.

b.) If for each positive integer n we have $f\left(1-\frac{1}{n}\right)=\left(1-\frac{1}{n}\right)^2$ then $f(z)=z^2$ on the unit disc.

c.) f cannot satisfy $f\left(\frac{1}{n}\right)=\frac{(-1)^n}{n}$ for each positive integer n.

d.) f cannot satisfy $f\left(\frac{1}{n}\right)=\frac{1}{n+1}$ for each positive integer n.

(80.) Let $f(z)=\frac{1+z}{1-z}$. Which of the following is/are true?

a.) f maps $\{|z|<1\}$ onto $\{\operatorname{Re}(z)>0\}$.

b.) f maps $\{|z|<1, \operatorname{Im}(z)>0\}$ onto $\{\operatorname{Re}(z)>0, \operatorname{Im}(z)>0\}$.

c.) f maps $\{|z|<1, \operatorname{Im}(z)<0\}$ onto $\{\operatorname{Re}(z)<0, \operatorname{Im}(z)<0\}$.

d.) f maps $\{|z|>1\}$ onto $\{\operatorname{Im}(z)>0\}$.

(81.) Let $f(z)=\dfrac{z-1}{\exp\left(\frac{2\pi i}{z}\right)-1}$. Then,

a.) f has an isolate singularity at $z=0$.

b.) f has a removable singularity at $z=1$.

c.) f has infinitely many poles.

d.) each pole of f is of order 1.

(82.) Let f be a meromorphic function on $\mathbb{C}$ such that $|f(z)|\geq|z|$ at each z where f is holomorphic. Then which of the following is/are true?

a.) The hypotheses are contradictory, so no such f exists.

b.) Such an f exists.

c.) There is a unique f satisfying the given conditions.

d.) There is an $A\in\mathbb{C}$ with $|A|\geq 1$ such that $f(z)=Az$ for each $z\in\mathbb{C}$.

(83.) Determine which of the following cannot be the class equation of a group a.) $10 = 1+1+1+2+5$.

b.) $4 = 1+1+2$.

c.) $8 = 1+1+3+3$.

d.) $6 = 1+2+3$.

(84.) Let A be a subset of $\mathbb{R}$ with more than one element. Let $a \in A$. If $A \setminus \{a\}$ is compact, then

a.) A is compact.

b.) every subset of A must be compact.

c.) A must be finite set.

d.) A is disconnected.

(85.) Let F and F' be two finite fields of order q and q' respectively. Then:

a.) F' contains a subfield isomorphic to F if and only if $q \le q'$.

b.) F' contains a subfield isomorphic to F if and only if q divides q'.

c.) If the g.c.d of q and q' is not 1, then both are isomorphic to subfields of some finite field L.

d.) Both F and F' are quotient rings of the ring $\mathbb{Z}[X]$.

(86.) Let R be a non-zero commutative ring with unity 1_R. Define the characteristic of R to be the order of 1_R in $(R,+)$ if it is finite and to be zero if the order of 1_R in $(R,+1)$ is finite. We denote the characteristic of R by char(R). In the following, let R and S be nonzero commutative rings with unity. Then

a.) Char(R) is always a prime number.

b.) if S is a quotient ring of R, then either char(S) divides char(R), or char(S) $=0$.

c.) if S is a subring of R containing 1_R then char$(S) =$ char(R).

d.) if char(R) is a prime number, then R is a field.

(87.) Which of the following subsets of $\mathbb{R}^2$ is/are NOT compact?

a.) $\{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 \le 1\}$.

b.) $\{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 \ge 1\}$

c.) $\{(x,y)\in\mathbb{R}^2 \mid x^2+y^2<1\}$

d.) $\{(x,y)\in\mathbb{R}^2 \mid x^2+y^2=1\}$

(88.) Let R be the ring obtained by taking the quotient of $(\mathbb{Z}/6\mathbb{Z})[X]$ by the principal ideal $(2X+4)$. Then

a.) R has infinitely many elements.

b.) R is a field.

c.) 5 is a unit in R.

d.) 4 is a unit in R.

(89.) Let $f(x)=x^3+2x^2+x-1$. Determine in which of the following cases f is irreducible over the field k.

a.) $k=\mathbb{Q}$, the field of rational numbers.

b.) $k=\mathbb{R}$, the field of real numbers.

c.) $k=F_2$, the finite field of 2 elements.

d.) $k=F_3$, the finite field of 3 elements

(90.) Let A and B be two disjoint nonempty subset of $\mathbb{R}^2$ such that $A\cup B$ is open in $\mathbb{R}^2$. Then,

a.) if A is open and $A\cup B$ is connected then B must be closed in $\mathbb{R}^2$.

b.) if A is closed, then B must be open in $\mathbb{R}^2$.

c.) if both A and B are connected, then $A\cup B$ must be disconnected.

d.) if $A\cup B$ is disconnected, then both A and B are open.

(91.) Let y be a nontrival solution of the boundary value problem

$$y''+xy=0,\ x\in[a,b],$$

$$y(a)=y(b)=0$$

then

a.) $b>0$.

b.) y is monotone in $(a,0)$ if $a<0<b$.

c.) $y'(a)=0$.

d.) y has infinitely many zeros in $[a, b]$.

(92.) Let $y : \mathbb{R} \to \mathbb{R}$ satisfy the initial value problem

$$y'(t) = 1 - y^2(t), t \in \mathbb{R},$$

$$y(0) = 0.$$

Then

a.) $y(t_1) = 1$ for some $t_1 \in \mathbb{R}$.

b.) $y(t) > -1$ for all $t \in \mathbb{R}$.

c.) y is strictly increasing in $\mathbb{R}$.

d.) y is increasing in $(0, 1)$ and decreasing in $(1, \infty)$.

(93.) Let $f(x) = e^x$ be approximated by Taylor's polynomial of degree n ate the point $x = \frac{1}{2}$ and on the entire interval $[0, 1]$. If the absolute error in this approximation does not exceed 10^{-2}, then the value of n should be taken as

a.) 0.

b.) 1.

c.) 2.

d.) 3.

(94.) The integral equation

$$\int_a^x K(x, y)\phi(y)\,dy = f(x)$$

With $K(x, x) \neq 0,$ for all x can be transformed to

$$\phi(x) + \int_a^x G(x, y)\phi(y)\,dy = g(x)$$

where for all x, y

a.) $K(x, y) = 1,\ g = f'$ and $G(x, y) = \frac{\partial K}{\partial x}(x, y)$.

b.) $K(x, x) = 1,\ g = f$ and $G(x, y) = 0$.

c.) $G(x, y) = \dfrac{1}{K(x,x)} \dfrac{\partial x}{\partial x}(x, y)$ and $g(x) = \dfrac{f'(x)}{K(x, x)}$.

d.) $G(x, y) = 1$ and $g(x) = f(x)$.

(95.) If the initial value problem for partial differential equation

$\dfrac{\partial u}{\partial t} + \dfrac{\partial^2 u}{\partial x^2} = 0;\ u(x, 0) = \sin(\pi x)$, has a solution of the form $u(x, t) = \phi(t)\sin(\pi x)$,

then

a.) ϕ is always negative.

b.) ϕ is always positive.

c.) ϕ is an increasing function.

d.) ϕ is a decreasing function.

(96.) Let $P(x, y)$ be a particular integral of the partial differential equation

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial x}{\partial y} = 2y - y^2,$$

Then $P(2, 3)$ equals

a.) 2.

b.) 8.

c.) 12.

d.) 10.

(97.) Let $H(\vec{q}, \dot{\vec{q}})$ denote respectively the Hamiltonian and Lagrangian of an autonomous system with $\vec{q}$ as generalized momentum and $\vec{q}$ the generalized coordinate vector. Then

a.) H remains conserved in the motion.

b.) H is simply the total energy of the system.

c.) $\vec{p}$ is constant if H is independent of $\vec{q}$.

d.) $\vec{p}$ is constant if L is independent of $\vec{q}$.

(98.) Consider a partition of the interval $[0,\,1]$ by points of subdivision $0 = x_0, x_1, \cdots, x_n = 1$ with each sub-interval of length h. Let m_i be the midpoint of the i^{th} sub-interval $[x_{i-1},\, x_i]$ and $f \in C^2([0,\,1])$. Then an error bound for the quadrature rule

$$\int f(x)\,dx \approx \sum_{i=1}^{n} f(m_i)\,h$$

is

a.) $|f''|_{\max} \dfrac{h^2}{2}$.

b.) $|f''|_{\max} \dfrac{h^3}{6}$.

c.) $|f''|_{\max} \dfrac{h^2}{24}$.

d.) $|f''|_{\max} \dfrac{h^4}{24}$.

(99.) Let $z = z(x,\, y)$ be a solution of

$$\frac{\partial z}{dx}\frac{\partial z}{\partial y} = 1$$

passing through $(0,\,0,\,0)$. Then $z(0,1)$ is

a.) 0.

b.) 1.

c.) 2.

d.) 4.

(100.) An extremal of the functional

$$J(y) = \int_a^b \sqrt{1 + |y'(x)|^2}\,dx$$

a.) is the straight line connecting $(a, y(a))$ and $(b, y(b))$.

b.) is a solution to the differential equation $y' = C\sqrt{1 + y'^2}$ for some constant C.

c.) is a solution to $y'' = 0$.

d.) does not exist.

(101.) Let $A=\begin{pmatrix}0&1&0\\0&0&1\\0&0&0\end{pmatrix}$ and $y=\begin{pmatrix}y_1(t)\\y_2(t)\\y_3(t)\end{pmatrix}$ satisfy

$$\frac{dy}{dt}=Ay;\ t>0;\ y(0)=\begin{pmatrix}1\\1\\1\end{pmatrix}$$

Then

a.) $y_1(t)=1+t+\frac{t^2}{2}$,

$y_2(t)=1+t,\ y_3(t)=1$.

b.) $y_1(t)=1+t$,

$y_2(t)=1+t+\frac{t^2}{2},\ y_3(t)=1$.

c.) $y_1(t)=1,\ y_2(t)=1+t$,

$y_3(t)=1+t+t^2/2$.

d.) $y_1(t)=e^{tA}y(0)$.

(102.) Let $f\in C^3([x_{-1},\ x_1])$ where $x_{-1}=x_0-h,\ \ x_1=x_0+h$ with $h>0,\ f(x_0)=f_0,\ \ f(x_j)=f_j$ for $j=-1,1$ and $f'(x_0)=f'_0$.

Then for some $\xi\in(x_{-1},x_1)$ we have

a.) $f'_0=\frac{f_1-f_0}{h}-\frac{h^2}{2}f'''(\xi)$.

b.) $f'_0=\frac{f_1-f_{-1}}{h}-\frac{h^3}{3}f'''(\xi)$.

c.) $f'_0=\frac{f_1-f_{-1}}{2h}-\frac{h^2}{6}f'''(\xi)$.

d.) $f'_0=\frac{f_1-f_{-1}}{2h}-\frac{h^3}{6}f'''(\xi)$.

(103.) Let $X_1,X_2,\cdots$ be independent and identically distributed standard normal random variables. Which of the following is true?

a.) $\dfrac{\sqrt{n}X_1}{\sqrt{X_1^2+\cdots+x_n^2}}$ has a t-distribution with $n-1$ degrees of freedom.

b.) $\dfrac{\sqrt{n}X_1}{\sqrt{x_1^2+\cdots+X_n^2}}$ has a t-distribution with n degrees of freedom.

c.) $\dfrac{\sqrt{n}X_1}{\sqrt{X_2^2+\cdots+X_{n+1}^2}}$ has t-distribution with $n-1$ degrees of freedom.

d.) $\dfrac{\sqrt{n}X_1}{\sqrt{X_2^2+\cdots+X_{n+1}^2}}$ has a t-distribution with n degrees of freedom.

(104.) Let $U_1, U_2 \cdots, U_n$ be i.i.d. random vector with common distribution $N_P(0, \Sigma), \Sigma = ((\sigma_{ij}))$. Define $S=\sum_{i=1}^{n} U_i U'_{j,}$ $S=((s_{ij}))$. Which of the following is/are true?

a.) $\sum_i \sum_j s_{ij} \sim$ constant $\cdot X_n^2$.

b.) $s_{11}-2s_{12}-s_{22} \sim$ constant $\cdot X_n^2$.

c.) $s_{11} \sim$ constant $\cdot X_n^2$.

d.) $s_{11}+s_{12}-2s_{12} \sim$ constant $\cdot X_n^2$.

(105.) Consider the following linear programming problem.

Maximize $z=2x_1+4x_2$

Subject to

$$x_1+2x_2 \le 5$$
$$x_1+x_2 \le 3$$
$$x_1, x_2 \ge 0.$$

a.) An optimum solution is $(x_1, x_2)=(1,2)$.

b.) An optimum solution is $(x_1, x_2)=(3,1)$.

c.) An optimum solution is $(x_1, x_2)=(0, 2.5)$.

d.) The objective function is unbounded.

(106.) Consider a queuing model with one service counter. The arrival and service processes are Poisson with rate λ and μ respectively. For $n = 0, 1, 2\cdots$ and $\mu > \lambda,$ let

$p_n = p$ {at any point of time there are n customers in the system}

$$= \left(1 - \frac{\lambda}{\mu}\right)\left(\frac{\lambda}{\mu}\right)^n.$$

Then the average queue length is

a.) $\frac{\lambda}{\mu - \lambda}$

b.) $\frac{\lambda}{\mu(\mu - \lambda)'}$

c.) $\frac{\lambda^2}{\mu(\mu - \lambda)'}$

d.) $\frac{\mu}{\mu - \lambda'}$

(107.) Suppose X is an exponential random variable with mean $1/\theta$. Due to round-off X is not observable, and Y defined as below is observed:

$Y = k$ if $k \le Y < k+1,\ k = 0,1,2\cdots$

Let $Y_1, Y_2, \cdots, Y_n$ be a random sample from the distribution of Y. Then a consistent estimator of θ based on $Y_1, Y_2, \cdots, Y_n$ is

a.) $\frac{n}{\sum_{i=1}^{n} Y_1}.$

b.) In $\left(1 + \frac{n}{\sum_{i=1}^{n} Y_i}\right).$

c.) In $\left(1 + \frac{n+1}{\sum_{i=1}^{n} Y_i}\right).$

d.) $\frac{n+1}{\sum_{i=1}^{n} Y_i}.$

(108.) Let $X_1, X_2, \cdots, X_n$ be a random sample from $N(\mu, \sigma^2)$ where μ is known. Let $C(k, \alpha)$ denote the quantile of order $1 - \alpha$ of X_k^2.

To test $H_1 : \sigma^2 \leq 1$ versus

H_1 : Reject H_0 if and only if

$\sum_{i-1}^{n} (x_i - \overline{X})^2 > c(n-1, \alpha)$

T_2 : Reject H_0 if and only if

$\sum_{i-1}^{n} (X_i - \mu)^2 > C(n, \alpha)$

Which of the following is/are true?

a.) T_1 is a UMP level α test.

b.) T_2 is a UMP level α test.

c.) Both T_1 and T_2 are level α tests.

d.) T_1 is a level α test but T_2 is not.

(109.) Let X be a geometric random variable with probability mass function given by

$P(X = k) = (1-p)^k p$ for $k \geq 0$ and $0 < p < 1$. For all $m, n \geq 1$ we have

a.) $P(X > m+n \mid X > m) = P(X \geq n)$.

b.) $P(X > m+n \mid X > m) = P(X > n)$.

c.) $P(X > m+n \mid X > m) = P(X < n)$.

d.) $P(X > m+n \mid X > m) = P(X \leq n)$.

(110.) Let $Y_1, Y_2, \cdots, Y_n$ be random variable such that $E(Y_i) = i\theta$, Var$(Y_i) = i^2\sigma^2$ and $C\,\text{ov}(Y_i, Y_j) = 0$ for all $1 \leq i, j \leq n, i \neq j$, where θ and σ^2 are unknown parameters. Consider the following two estimators of θ :

$$T_1 = \frac{1}{n}\sum_{i=1}^{n}\frac{Y_i}{i},$$

$$T_2 = \frac{6}{n(n+1)(2n+1)}\sum_{i=1}^{n} iY_i.$$

Which of the following statement(s) is(are) true?

a.) T_1 is the best linear unbiased estimator of θ .

b.) T_2 is the ordinary least square estimator of θ.

c.) $Var(T_1) = \frac{\sigma^2}{n}$.

d.) An unbiased estimator of σ^2 is

$$\frac{1}{n-1}\left[\sum_{i=1}^{n}\frac{Y_i^2}{i^2} - \frac{1}{n}\left(\sum_{i=1}^{n}\frac{Y_i}{i}\right)^2\right].$$

(111.) Let $X_1, X_2, \cdots, X_m$ and $Y_1, Y_2, \cdots, Y_n$ be two independent random samples form two continuous distribution F_1 and F_2 respectively. Define

$R_i =$ Rank(X_i), $i = 1, 2, \cdots, m$ and

$S_j =$ Rank$(Y_j), = 1, 2, \cdots, n$

in the combined sample

$(X_1, X_2, \cdots, X_m, Y_1, Y_2, \cdots, Y_n)$.

Also define

$$I(U > V) = \begin{cases} 1 & \text{if } U > V \\ 0 & \text{if } U \le V \end{cases}.$$

Which of the following test statistic is/are distribution free under $H_0 : F_1(x) = F_2(x)$ for all x ?

a.) $\sum_{i=1}^{m} R_i$

b.) $\sum_{j=1}^{n} S_j$.

c.) $\sum_{i=1}^{m}\sum_{j=1}^{n} I(X_i > Y_j)$.

d.) $\sum_{i=1}^{m}\sum_{j=1}^{n} I(S_j > R_i)$.

(112.) Let X and Y be random variables with $EX^2 < \infty$. Then, we can conclude that

a.) Var $(X) \ge$ Var $(E(X \mid Y))$.

b.) Var $(X) \geq E(E(X|Y))$.

c.) Var $(E(X|Y)) \geq E(\text{Var}(X|Y))$.

d.) $E(\text{Var}(X|Y)) \geq \text{Var}(E(X|Y))$.

(113.) An incomplete block design involving 5 treatments, labeled $1,2,\cdots,5$ and two blocks has the following block contents:

Block I $(1,2,3)$;
Block II $(1,4,5)$.

For $i=1,2,\cdots,5$ let t_i be the effect of the i^{th} treatment. Which of the following statement(s) is (are) true?

a.) The design is disconnected.

b.) All contrasts $t_i - t_j$

$(i, j = 1,2,\cdots,5,\ i \neq j)$ are estimable.

c.) The variances of the best linear unbiased estimators of all contrasts $t_i - t_j$ are the same, assuming that all observation have the same variance.

d.) The Variance of the best linear unbiased estimators of $t_i - t_j$ is either $2\sigma^2$ or $4\sigma^2$, where σ^2 is the variance of an observation.

(114.) Let $X_1, X_2, \cdots, X_n$ be a random sample from Uniform $(\theta, 3\theta), \theta > 0$. Let $X_{(n)} = \max\{X_1, X_2, \cdots, X_n\}$ and $X_{(1)} = \min\{X_1, X_2, \cdots, X_n\}$.

Which of the following is/are true?

a.) $\dfrac{X_{(n)} - X_{(1)}}{2}$ is unbiased for θ.

b.) $X_{(1)}, X_{(n)}$ is complete sufficient for θ.

c.) $\dfrac{X_{(n)}}{X_{(1)} + X_{(n)}}$ is an ancillary statistic.

d.) $\dfrac{1}{2m}\sum_{i=1}^{m} X_i$ is unbiased for θ.

(115.) A sample of size $m(n \geq 2)$ is drawn from a finite population of N units by probability proportional to size sampling with selection probability p_i.

$\left(1 \le i \le N,\, 0 < p_i < 1,\, \sum_{i=1}^{N} p_i = 1\right)$.

Let $T = \frac{1}{n}\sum \frac{p_i}{p_i}$

where y_i is the value of a study variable for the i^{th} unit the sum extends over the units inclided in the sample. Which of the following statement(s) is(are) true?

a.) T is an unbiased estimator of the population total $\sum_{i=1}^{N} y_i$.

b.) mT is an unbiased estimator or the population total $\sum_{i=1}^{N} y_i$.

c.) The variance of T reduces to 0 if $p_i = 1/N$ for all i, $1 \le i \le N$.

d.) The variance of T reduces to 0 if y_i is proportional to p_i for all i, $1 \le i \le N$.

(116.) Suppose T denote the survival time of a component having probability density function $f(t)$. T has an exponential distribution if and only if

a.) $\frac{f(t)}{p\{T > t\}}$ is independent of t for all $t > 0$

b.) $P\{T > t + s\} = P\{T > t\}P\{T > s\}$ for all, $s > 0$.

c.) $P\{T < t + s\} = P\{T < t\}P\{T < s\}$ for all t, $s > 0$.

d.) $P\{T < t + s\} = P\{T > t\}P\{T < s\}$ for all t, $s > 0$.

(117.) Let $X \sim N_p(\mu, I)$ and $B_{p\times p}$ be any real symmetric matrix of rank $k \le p$ such that $B\mu = 0$ and $B^2 = B$. Then the probability distribution of $X'BX$ is

a.) Wishart.

b.) $\chi_k^2 - 1$.

c.) χ_k^2.

d.) The same as that of $\sum_{i=1}^{k} Z_i^2$ where Z_i are independent $N(0, 1)$.

(118.) Let $X_1, X_2, \cdots$ be a Markov chain. For $n \ge 1$ and for any two states k and l, let

$p_{kl}^{n} = P\left(X_{m\times n} = l \mid X_m = k\right)$ for all $m \ge 1$.

Suppose $p_{ij}^{n} > 0$ and $p_{ji}^{m} > 0$ for some states i and j and for some $n, m \ge 1$. Identify the correct statements.

a.) If i is transient than j is transient.

b.) $d(i)=d(j)$ where for any state $l, d(l)$ denotes the period of state l.

c.) If the Markov chain is aperiodic then $\lim_{n\to\infty} p_{ii}^{n} = \lim_{n\to\infty} p_{jj}^{n}$.

d.) If the period of state j is 1, then $\lim_{n\to\infty} p_{ij}^{n} = \lim_{n\to\infty} p_{jj}^{n}$.

(119.) Let $X_1, X_2, \cdots$ be independent and identically distributed random variables each having a uniform distribution on $[-1,1]$. For $n \geq 1$, let $S_n = \sum_{i=1}^{n} X_i$ and let $Z_n = S_n / n^p$ for some $p > 0$.

Then, as $n \to \infty$.

a.) $Z_n \to 0$ almost surely for $p \geq 1$.

b.) $Z_n \to 0$ in probability for $\frac{1}{2} < p < 1$.

c.) Z_n converges in distribution to a non-degenerate random variable if $p = \frac{1}{2}$.

d.) $Z_n \to \infty$ almost surely for $p < \frac{1}{2}$.

(120.) Let X_1 and X_2 be independent random variables with cumulative distribution functions (cdf) F_1 and F_2 respectively. Let G be the cdf of $X_1 + X_2$ and H be the cdf of $X_1 X_2$. Identify the correct statements.

a.) If F_1 is a continuous function then so is G.

b.) If F_1 is a continuous function then so is H.

c.) If F_1 is a step function then so is G.

d.) If F_1 is step function then so is H.